# Probability and Probability Distributions

## I. Probability

### A. Introduction: Lotteries as losing bets ("voluntary taxes")

Sucker Bet: wager in which expected return is significantly lower than the wager.

EXPECTED RETURN = winning prize * P(win) – cost of wager * P(lose).

### B. Preliminaries -- see Zar chapter 5

Counting Possible Outcomes; Permutations; Combinations

Prob. of an Event: rel. frequency = frequency of event / total number all events    Range: [0,1]

Adding Probabilities

Mutually exclusive events (e.g., heads/tails): P(A or B) = P(A) + P(B)

Not mutually exclusive (i.e., intersecting): P(A or B) = P(A) + P(B) – P(A and B)

Multiplying Probabilities

P(A and B) = P(A)*P(B)

### C. Probability density function (pdf): P(X = x), i.e., prob that variate has value x

For continuous distribution [since P(X = x) = 0 ], usually $P(x_a \le X \le x_b)$

### D. Cumulative distribution function (cdf): $P(X \le x)$, i.e., prob that variable ≤ x

## II. Probability Distributions

http://www.itl.nist.gov/div898/handbook/eda/section3/eda366.htm

### A. Continuous Distributions

1. Uniform Distribution

pdf: $f(x) = \dfrac{1}{B - A}$ for $A \le x \le B$

i.e., standard uniform pdf: $f(x) = 1$ for $0 \le x \le 1$

graph: horizontal line at P = 1

cdf: $f(x) = x$ for $0 \le x \le 1$

mean: $(A + B)/2$    SD: $\sqrt{\dfrac{(B-A)^2}{12}}$    CV: $\dfrac{B - A}{\sqrt{3}(B + A)}$

2. Normal Distribution: from Central Limit Theorem

pdf: $f(x) = \dfrac{e^{-(x-\mu)^2/(2\sigma^2)}}{\sigma\sqrt{2\pi}}$

pdf of standard normal ($\mu$ = 0, $\sigma$ = 1): $f(x) = \dfrac{e^{-x^2/2}}{\sigma\sqrt{2\pi}}$

mean: $\mu$    SD: $\sigma$    CV: $\mu/\sigma$

3. t Distribution

leptokurtic (heavy tails) relative to Normal distribution

mean: $\mu$    SD: $\sqrt{\dfrac{\nu}{\nu - 2}}$

4. Chi-square Distribution
   sum of ν independent standard normal distributions, each squared

   pdf: $f(x) = \dfrac{e^{-x/2} x^{\frac{\nu}{2}-1}}{2^{\nu/2}\Gamma\left(\frac{\nu}{2}\right)}$ for $x \geq 2$, $\Gamma(a) = \int_0^\infty t^{a-1} e^{-t}\, dt$

   mean: $\nu$ SD: $\sqrt{2\nu}$ CV: $\sqrt{2/\nu}$

5. F Distribution: ratio of two Chi-square distributions

   mean: $\dfrac{\nu_2}{(\nu_2 - 2)}$, $\nu_2 > 2$ CV: $\sqrt{\dfrac{2(\nu_1 + \nu_2 - 2)}{\nu_1(\nu_2 - 4)}}$, $\nu_2 > 4$

## B. Discrete Distributions

1. Binomial Distribution: 2 mutually exclusive outcomes per event

   probability mass function: $p(x, p, n) = \binom{n}{x} p^x (1-p)^{n-x}$, for $x$ = 0, 1, 2, …, n

   where $\binom{n}{x} = \dfrac{n!}{x!(n-x)!}$

   cumulative probability function: $F(x, p, n) = \sum_{i=0}^{x} \binom{n}{x} p^i (1-p)^{n-i}$

   mean: $np$ SD: $\sqrt{np(1-p)}$ CV: $\sqrt{\dfrac{1-p}{np}}$

2. Poisson Distribution: e.g., number of events w/in time (or space) interval

   probability mass function: $p(x, \lambda) = \dfrac{e^{-\lambda}\lambda^x}{x!}$, for $x$ = 0, 1, 2, …

   mean: $\lambda$ SD: $\sqrt{\lambda}$ CV: $1/\sqrt{\lambda}$

# III. **Conditional Probability**

Bayes Theorem: P(B|A) = P(AB) / P(A)